ACKNOWLEDGMENTS

One of us (R. B. W.) wishes to express his appreciation to Walter B. Hewlett and Michael J. Y. Williams for their generous assistance with the numerical computations presented in this paper.

*Work supported by the Advanced Research Projects Agench through the Center for Materials Research at Stanford University, Stanford, Calif. 94305.

†National Science Foundation Predoctoral Fellow.

[1]T. Wolfram and J. Callaway, Phys. Rev. 127, 1605 (1962).

[2]C. Haas, Phys. Rev. 168, 531 (1968).

[3]F. Rys, J. Helman, and W. Baltensperger, Physik Kondensierten Materie 6, 105 (1967).

[4]R. M. White, Phys. Rev. Letters 23, 858 (1969).

[5]R. M. White and R. B. Woolsey, Phys. Rev. 176, 908 (1968).

[6]S. V. Vonsovskii and Y. A. Izyumov, Usp. Fiz. Nauk 78, 3 (1962) [Soviet Phys. Usp. 5, 723 (1963)].

[7]H. S. D. Cole and R. E. Turner, Phys. Rev. Letters 19, 501 (1967).

[8]See, for example, D. Pines, in *Polarons and Excitons*, edited by C. G. Kuper and G. D. Whitfield (Oliver and Boyd, London, 1963).

[9]S. Methfessel and D. C. Mattis, in *Handbuch der Physik*, edited by S. Flügge (Springer, Berlin, 1968), Vol. 18/1, p. 484.

[10]B. Giovannini, M. Peter, and S. Koide, Phys. Rev. 149, 251 (1966).

[11]R. B. Woolsey, Ph.D. dissertation, Stanford University, Stanford, Calif., 1970 (unpublished).

[12]H. S. D. Cole, Phys. Letters 30A, 114 (1969).

[13]P. N. Butcher, Rept. Progr. Phys. 30, 97 (1967).

[14]A. A. Abrikosov, L. P. Gor'kov, and I. Y. Dzyaloshinskii, in *Quantum Field Theoretical Methods in Statistical Physics*, edited by D. ter Harr (Pergamon, New York, 1965), p. 188.

PHYSICAL REVIEW B VOLUME 1, NUMBER 11 1 JUNE 1970

COMMENTS AND ADDENDA

The Comments and Addenda section is for short communications which are not of such urgency as to justify publication in Physical Review Letters and are not appropriate for regular Articles. It includes only the following types of communications: (1) comments on papers previously published in The Physical Review or Physical Review Letters; (2) addenda to papers previously published in The Physical Review or Physical Review Letters, in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section may be accompanied by a brief abstract for information-retrieval purposes. Accepted manuscripts will follow the same publication schedule as articles in this journal, and galleys will be sent to authors.

Composite Ising Lattices with Unequal Spins*

Robert H. T. Yeh
Department of Physics and Astronomy, State University of New York, Buffalo, New York 14214
(Received 30 January 1970)

The phase-transition problem of two-component composite Ising lattices with unequal spins in zero magnetic field is solved within the Bragg-Williams approximation.

We shall generalize our previous treatment of composite Ising lattices[1] to the case where spins of different components of the composite may have different magnitudes. We shall restrict ourselves to the two-component case with zero external field, although the method can easily be generalized to three or more components cases. Unless otherwise specified, we shall follow the notations of I.

Consider a two-component lattice with interaction constants ϵ_1, ϵ_2, ϵ_3,; structure parameters u_1, u_2, u_3, v_1, v_2; and spin magnitudes s_1, s_2. With zero external field, the partition function of this lattice is equal to the partition function of a lattice with same structure, but different interaction constants ϵ_1', ϵ_2', ϵ_3' and spin magnitudes $s_1' = s_2' = 1$, and

$$\epsilon_1' = \epsilon_1 s_1^2, \quad \epsilon_2' = \epsilon_2 s_2^2, \quad \epsilon_3' = \epsilon_3 s_1 s_2 \ . \tag{1}$$

So with this transformation and the results of I, we can easily write down the equations for composite Ising lattices with unequal spins. The magnetization per site is given by

$$M/N = 2v_1 s_1 L_1 + 2v_2 s_2 L_2 \ , \tag{2}$$

with

$$2L_1 = \tanh\left[4\beta\gamma\epsilon_1' L_1\left(\frac{u_1}{v_1}\right) + 2\beta\gamma\epsilon_3' L_2\left(\frac{u_3}{v_1}\right)\right] ,$$
$$2L_2 = \tanh\left[4\beta\gamma\epsilon_2' L_2\left(\frac{u_2}{v_2}\right) + 2\beta\gamma\epsilon_3' L_1\left(\frac{u_3}{v_2}\right)\right] . \tag{3}$$

A concrete example is given in Fig. 1, with $v_1 = v_2 = \frac{1}{2}$, $u_1 = u_2 = \frac{1}{2}$, $u_3 = \frac{1}{8}$, $k_B T_{c1} = \gamma\epsilon_1$; the solid line represents the solution of Eq. (2) for $s_1 = 1$, $s_2 = 2$, $\epsilon_2 = \frac{1}{8}\epsilon_1$, and $\epsilon_3 = (\epsilon_1\epsilon_2)^{1/2}$. The dashed lines represent magnetization curves of corresponding single-component systems. As conjectured in I, we see that the composite system has a higher value of M/N in the temperature range $0.42 < T/T_{c1} < 0.63$.

On the experimental side, there is no evidence of definite correlation between coupling strength and spin magnitude in ferromagnetic insulators,[2] hence the above phenomena are not entirely unrealistic, although this is by no means an easy experiment.[3]

For two-component systems, one can easily establish that $2u_1 + u_3 = v_1$ (or equivalently $2u_2 + u_3 = v_2$). As a special case, this implies that if $v_1 = v_2$, then $u_1 = u_2$ and vice versa. This is, in general, not true for lattices with more than two components. As a counter example, consider a unit cell of three components *A B B C A C*. Here $v_1 = v_2 = v_3$, but u's are not identical.

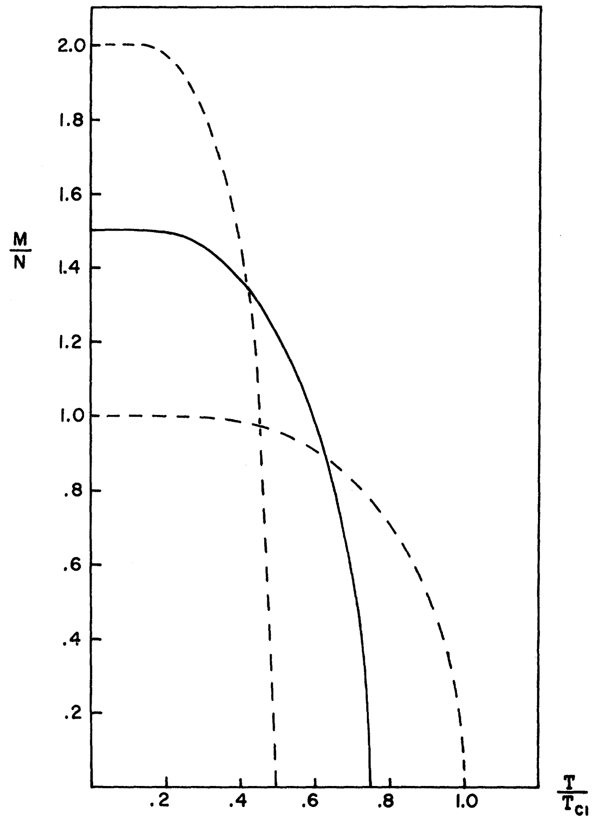

FIG. 1. Magnetization curves of composite Ising lattice with unequal spins. $v_1 = v_2 = \frac{1}{2}$, $u_1 = u_2 = \frac{1}{2}u_3 = \frac{1}{8}$, $k_B T_{c1} = \gamma\epsilon_1$. Solid line represents the case of $s_1 = 1$, $s_2 = 2$, $\epsilon_2 = \frac{1}{8}\epsilon_1$, and $\epsilon_3 = (\epsilon_1\epsilon_2)^{1/2}$. The dashed lines represent magnetization curves of corresponding single-component systems.

---

*Work supported by a faculty research fellowship from the Research Foundation of State University of New York.

[1]R. H. T. Yeh, Phys. Rev. B 1, 1180 (1970). We shall refer to this paper as I.

[2]For example, CrO_2 has $T_c = 400\,°K$ and $2\mu_B$ (Bohr magneton) per Cr^{4+} ion for saturation magnetization. $CrBr_3$ has $T_c = 37\,°K$ and $3\mu_B$ per Cr^{3+} ion for saturation magnetization. For a summary of experimental results, see *Magnetism III*, edited by G. T. Rado and H. Suhl (Academic, New York, 1963), Chap. 2.

[3]R. H. T. Yeh, Phys. Status Solidi 30, K45 (1968).